Unit 2

Boolean Algebra

Outline

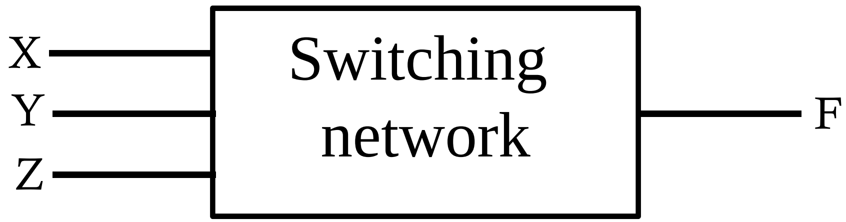
- Basic operations
- Boolean expression and Truth table
- Basic theorems
- Advanced theorems
- Multiplying out and factoring
- DeMorgan’s Laws and duality

Basic Operations (1/3)

Boolean Variables: X, Y, Z … takes “0”, “1”

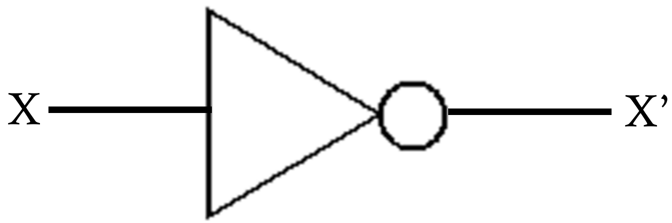
“L”, “H”

“F”, “T”



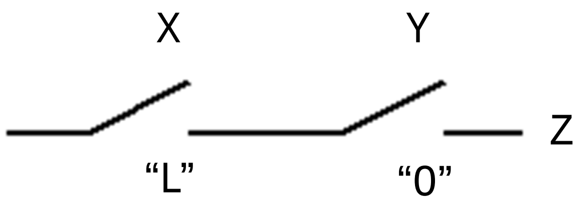
Complement (NOT or Inverter):

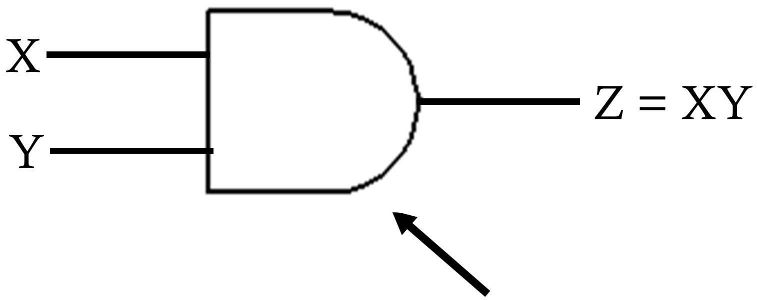
$0 \leftrightarrow 1 \quad x \leftrightarrow x'$ (or $x \leftrightarrow \bar{x}$)



Basic Operations (2/3)

$$AND \quad 0 \cdot 0 = 0 \quad 0 \cdot 1 = 0 \quad 1 \cdot 0 = 0 \quad 1 \cdot 1 = 1$$





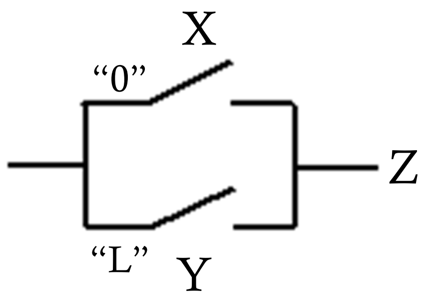
AND Gate

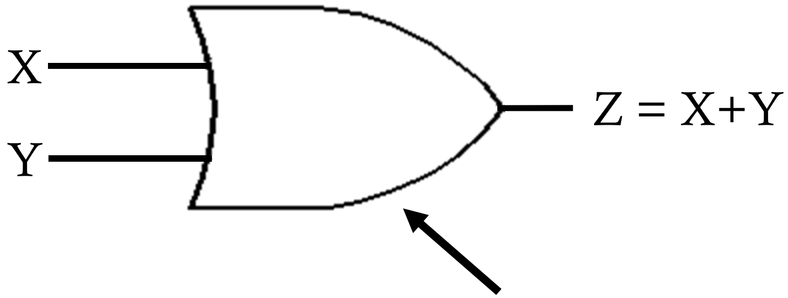
Truth Table

X	Y	$Z = X \cdot Y$
0	0	0
0	1	0
1	0	0
1	1	1

Basic Operations (3/3)

$$OR \quad 0+0=0 \quad 0+1=1 \quad 1+0=1 \quad 1+1=1$$





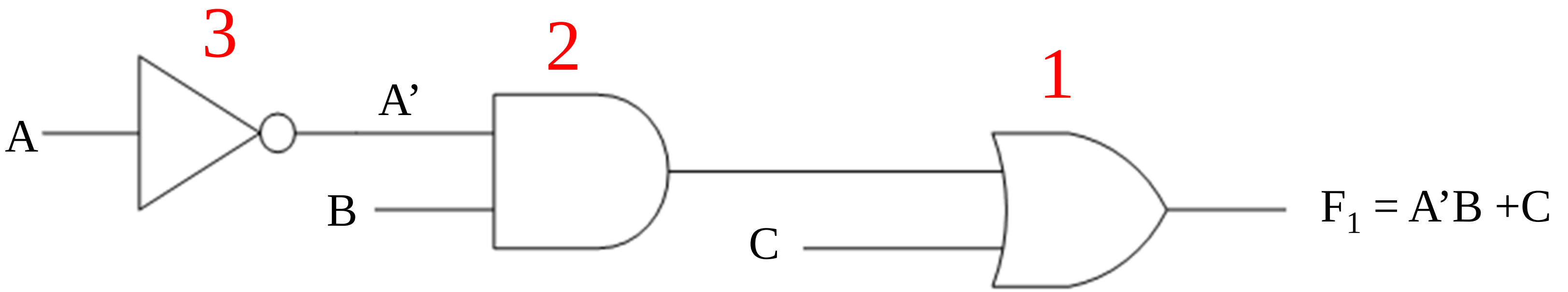
OR gate

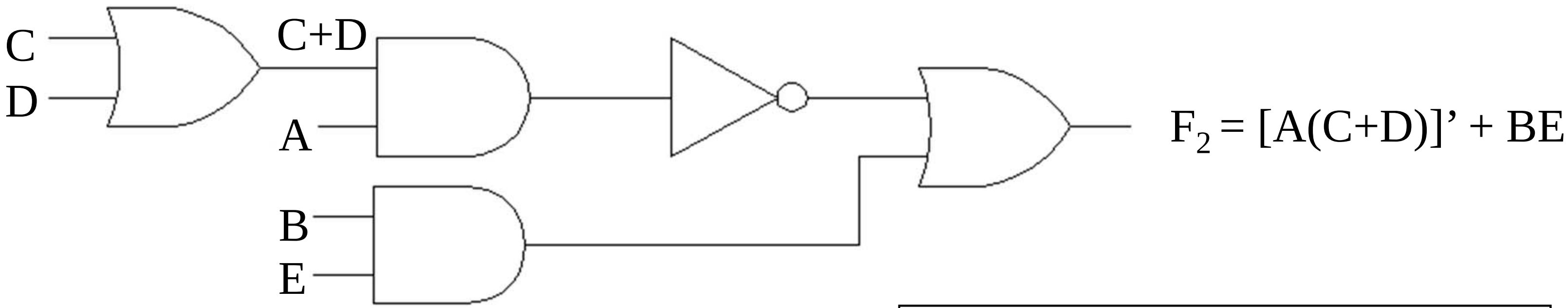
Truth Table

X	Y	$Z = X + Y$
0	0	0
0	1	1
1	0	1
1	1	1

Boolean Expression and Truth Table (1/2)

$$Boolean\ \ Expression: \quad F_1 = A'B + C \quad F_2 = [A(C+D)]' + BE$$





$$if \quad A = B = C = 1, D = E = 0$$

$$\Rightarrow \quad F_1 = 0 \cdot 1 + 1 = 1$$

$$F_2 = [1(1+0)]' + 1 \cdot 0 = 0$$

Boolean Expression and Truth Table (2/2)

Truth table size = $2^3 = 2^n$

A	B	C	B'	AB'	$AB'+C$	$A+C$	$B'+C$	$(A+C)(B'+C)$
0	0	0	1	0	0	0	1	0
0	0	1	1	0	1	1	1	1
0	1	0	0	0	0	0	0	0
0	1	1	0	0	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	0	1	0	0
1	1	1	0	0	1	1	1	1

Basic Theorems

$$x + 0 = x \qquad x \cdot 1 = x$$

$$x + 1 = 1 \qquad x \cdot 0 = 0$$

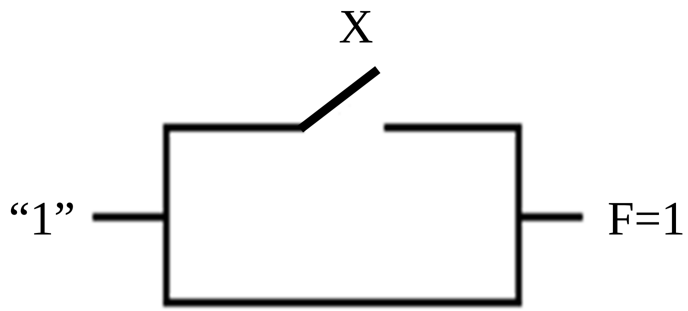
$$x + x = x \qquad x \cdot x = x$$

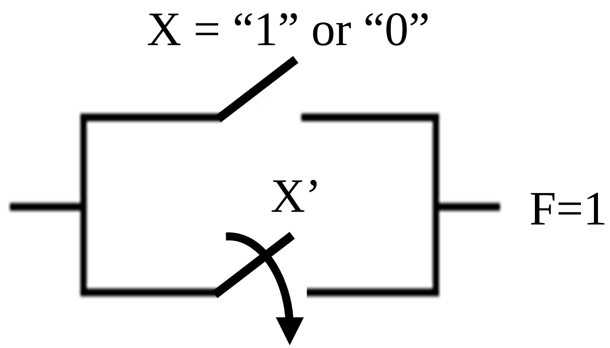
$$(x')' = x$$

$$x + x' = 1 \qquad x \cdot x' = 0$$

$$\Rightarrow \quad (AB'+D)E + 1 = 1$$

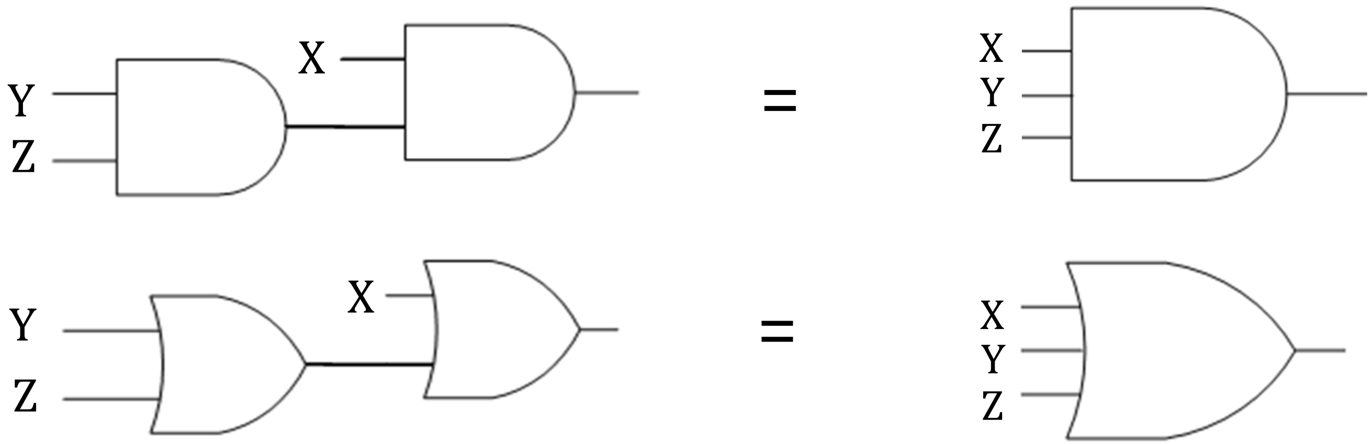
$$(AB'+D)(AB'+D)' = 0$$





Advanced Theorems (1/4)

Commutative Law		$xy = yx$	$x + y = y + x$
Associative Law	$\begin{cases} (xy)z = x(yz) = xyz \\ (x+y)+z = x+(y+z) = x+y+z \end{cases}$		



$$\Rightarrow \quad xyz = 1 \quad \Rightarrow \quad x = 1 = y = z$$

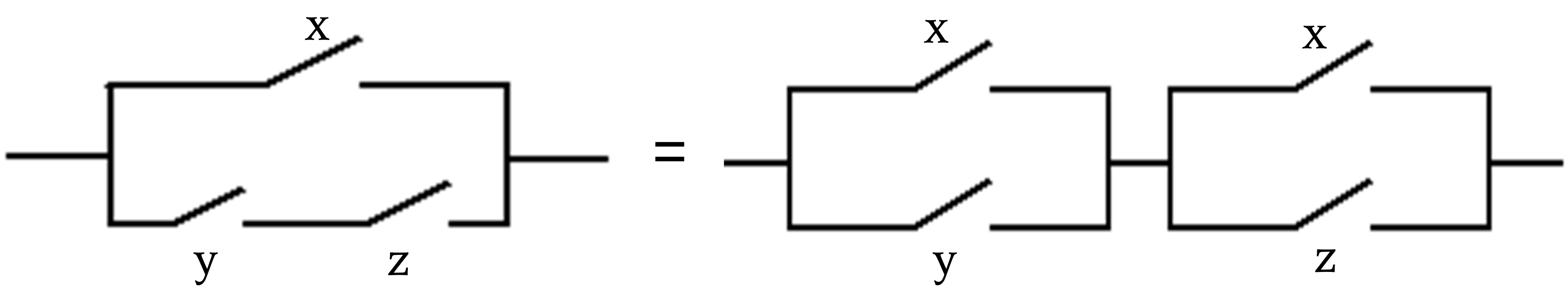
$$x + y + z = 0 \quad \Rightarrow \quad x = 0 = y = z$$

Advanced Theorems (2/4)

$$\left.\begin{aligned} x(y+z) &= xy+xz \\ x+yz &= (x+y)\cdot(x+z) \end{aligned}\right\} \quad \textit{Distributive Law}$$

Only valid for Boolean Algebra

$$\begin{aligned} pf:\ \text{RHS}\quad (x+y)\cdot(x+z) &= x\cdot(x+z)+y(x+z) = x\cdot x+xz+yx+yz \\ &= x+xz+xy+yz = x(1+z+y)+yz \\ &= x+yz \end{aligned}$$



Advanced Theorems (3/4)

$xy + xy' = x$	$(x+y)(x+y') = x \cdots 2$
$x + xy = x \cdots 1$	$x(x+y) = x$
$(x+y')y = xy$	$xy' + y = x + y$

$$pf \quad 1)\ x + xy = x(1+y) = x$$

$$2)\ (x+y)(x+y') = x + yy' = x + 0$$

Advanced Theorems (4/4)

- Ex1: $Z = A'BC + A' = A'(1 + BC) = A'$

- Ex2: $Z = [A + B'C + (D + EF)][A + B'C + (D + EF)']$

  $= [X + Y][X + Y']$

  $= A + B'C$

- Ex3: $Z = (AB + C)(B'D + C'E') + (AB + C)'$

  $= (X)(Y) + (Y)'$

  $= X + Y'$

  $= (B'D + C'E') + (AB + C)'$

Multiplying Out and Factoring (1/5)

Sum of Product form:

1. AB’ + CDE + AC’E’
2. A + B + C’ + DE’
3. ABC + AB’C + AB’C’
4. AB + DEFG + H

(A+B)CD+EF is not a sum of product (SOP) form

Multiplying Out and Factoring (2/5)

Multiplying Out: Get expressions to be SOP form

When multiplying out:

Use (A+B)(A+C) = A + BC

$$Ex.\quad (A + \underbrace{BC}_{x})(A + \underbrace{D+E}_{y}) = A + xy$$
$$= A + BC(D+E)$$
$$= A + BCD + BCE$$

Multiplying Out and Factoring (3/5)

Product of Sum form (POS)

$$1.\ AB'C(D'+E)$$

$$2.\ (A+B)(C+D+E)F$$

$$3.\ (A+B')(C+D'+E)(A+C'+E')$$

Factoring: Get expression to be POS form

Use $x + yz = (x+y)(x+z)$

Multiplying Out and Factoring (4/5)

POS form: Example

$$Ex:1.\ A+BCD' = (A+B)(A+C)(A+D')$$

$$2.\ AB'+C'D = (AB'+C')(AB'+D)$$

$$= (A+C')(B'+C')(A+D)(B'+D)$$

$$3.\ C'D+C'E'+G'H = C'(D+E')+G'H$$

$$=(C'+G'H)(D+E'+G'H)$$

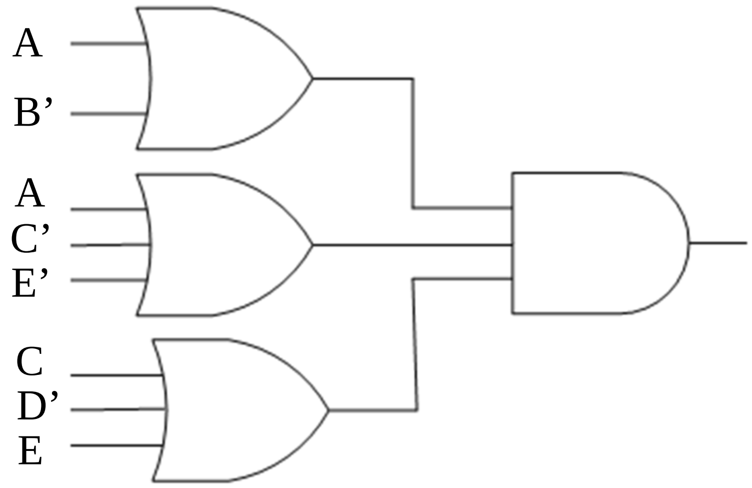
$$=(C'+G')(C'+H)(D+E'+G')(D+E'+H)$$

Multiplying Out and Factoring (5/5)

2-level realization

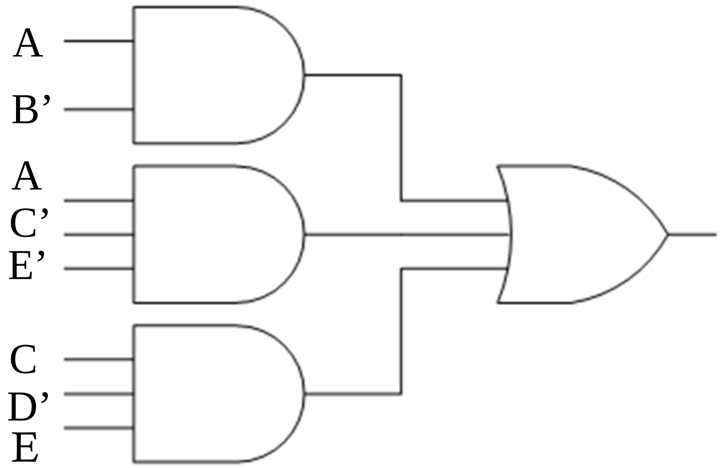
POS

$$(A+B')(A+C'+E')(C+D'+E)$$



SOP

$$AB'+AC'E'+CD'E$$



(A', B', C' use inverter)

DeMorgan's Law and Duality (1/3)

$$(x+y)' = x'y' \qquad (xy)' = x'+y'$$

$$(x+y+z+\cdots)' = x'y'z'\cdots \qquad (xyz\cdots)' = x'+y'+z'+\cdots$$

$$\bullet \leftrightarrow + \quad , \quad A \leftrightarrow A'$$

Proved by Truth table

1. $[(A'+B)C']' = (A'+B)'+C = AB'+C$

2. $[(AB'+C)D'+E']' = [(AB'+C)D']' \bullet E = [(AB'+C)'+D]E$
$= [(AB')'C'+D]E = [(A'+B)C'+D]E$
$= A'C'E + BC'E + DE$

DeMorgan's Law and Duality (2/3)

General Form: (one-step rule)

$$[f(x_1,\ x_2,\ \cdots,\ x_n,\ 0,\ 1,\ +,\ \bullet)]' = f(x_1',\ x_2',\ \cdots,\ x_n',\ 1,\ 0,\ \bullet,\ +)$$

Example

$$[(a'b+c')(d'+ef')+gh+w]' = [(a+b')c+d(e'+f)](g'+h')w'$$

DeMorgan's Law and Duality (3/3)

General Form

$$[f(x_1,\ x_2,\ \cdots,\ x_n,\ 0,\ 1,\ +,\ \bullet)]^D = f(x_1,\ x_2,\ \cdots,\ x_n,\ 1,\ 0,\ \bullet,\ +)$$

$AND \leftrightarrow OR \qquad 0 \leftrightarrow 1$

$$Ex.\quad F = ab'+c+0\cdot d'(1+e)$$
$$F^D = (a+b')c(1+d'+0\cdot e)$$

1. form F' 2. $a \leftrightarrow a'$

$$F' = (a'+b)c'\cdot(1+d+0\cdot e')$$
$$F^D = (a+b')c(1+d'+0\cdot e)$$

Property

$$F = G \quad \Rightarrow \quad F^D = G^D$$

$$Ex.\quad (x+y')y = xy \xrightarrow{Duality} x\cdot y'+y = x+y$$